Visual Simulation of Crack Pattern Based on 3D Surface Cellular Automaton

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Abstract

This article describes a method for modeling the propagation of cracks on any 3D surface. Taking a previous cellular automata model as basis [13], this method allows about any type of cracks on any type of triangulated 3D object. Our model's main advantage is that it proposes a semi-physical solution, making it at the same time user controllable and easily extensible. After summarizing works in the literature, we make a brief and simple description of what physically are cracks and how they are generated. Based on this idea, we detail our model of crack propagation. We first introduce the general development of cracks. We then propose our original model of spectrum stress. This is followed by the description of the mutual interaction between cracks and stresses. Finally, a set of graphical examples, with their respective parameters, concludes this paper.

Keywords: Cellular automata, multi-layer rendering, weathering, fracture, cracking, and deformation.

1 Introduction.

Nowadays one of the main topics in the field of Computer Graphics (CG) is the realization of realistic texturing on the surface of 3D objects. In this section, we first summarize the literature of the domain, then deduce the improvements to be done –the basis for our model—and present how to achieve it.

The background of this research can generally be subdivided into two main domains. On the one hand, physical approaches propose realistic models but are largely restricted by huge computational times and (often) poor resulting images. We only cite as example some of the best articles on this field: modeling inelastic deformation [27], simulation of 3D crack [14], generation of crack patterns with a physical model [15], experimental study on mud crack patterns [17], animation of fracture by physical modeling [19], fracture in microsphere monolayers studies by experiment and computer simulation [23], and very recently in SIGGRAPH'99, graphical modeling and animation of brittle fracture [20]. Describing the contrast between all these articles and the present manuscript would be terribly long and tedious. However, and for clarity reason, we would like to point out the main differences with this recent and famous article [20]. Object's fracture in 3D is certainly much more impressive than the propagation of cracks on 3D surface. Nevertheless, it doesn't serve the same purposes. In particular, we describe that in our model the range of materials and generated crack patterns is multiple, that cracks start, evaluate, interact, and stop through times, that their velocities and curvature depends of their history, and especially that no external force is needed to generate them.

On the other hand, other models often called "faking models" (see [1] for an excellent explanation of the need of "fake" in CG), such as texture mapping techniques, are very convenient, as they are easy to implement, quick to compute, and give impressive results. But mappings also generate many limitations as they require large texturing libraries, mapping orientation is often difficult, scaling and resolution problems arise, texturing continuities at edges are most of the time inconsistent, and real 3D texturing extension is about impossible. As example, one of the most recent publication on "ultra-realistic" texturing using mapping texture proposes a general approach on "reflectance and texture of real-world surface" [6]. Still faking approaches are simple and indeed efficient models to simulate continuous crack using slopes attraction as showed by "a behavioral model of cracks and its applications to CG" [4].

CG requires *automatic textural* effects that do not exhibit these limitations and that can be intuitive enough to be implemented and computed. Unfortunately, the literature in this particular area remains very limited. Models showing very interesting results for automatic texturing simulation –e.g. metallic patina [8], Dust accumulation [16], surface imperfection [31]—have only been published since about 1994. More recently, we proposed a more general approach introduced for 2.5D in [12] and detailed in 3D in [13] using 3D surface Cellular Automata (CA) (see [3, 5, 21] for CA theories). CA can be very useful in visual simulation because we can identify the object not only as a set of polygons covering the visible surface, but as a set of material layers –themselves subdivided into regular cells with independent behaviors. This allows the object to become dynamic: **it becomes *alive* through time**.

Articles or books referring to CA in CG are relatively easy to find, e.g. [10, 11, 22, 24, 25, 26, 29], but the ones dealing specifically on surface CA for generating automatic texturing are fairly rare [29]. Thus, this field remains largely open for new discoveries.

For more explanations and classification of most of the articles previously sited, please refer to [13].

Based on our 3D CA model we present a solution for simulating realistic propagation of various types of cracks. The crack propagation is automatically generated using an original "intuitive-physical" approach. "Physical", because seen from a CG scientist's point of view this model sounds very theoretical, and "intuitive" because in reality our model is very far from being realistic –from a physicist's point of view, of course—and therefore remains *intuitive*.

2 General concepts of stresses and cracks.

Before explaining anything about our model for generating the propagation of cracks through 3D multi-layer CA, we review some general and simple considerations: What are cracks? Why do cracks happen? What are stresses or stress-fields? How can they be represented? And what is one of the most important achievements of this paper?

A crack is the systematic breaking of material liaisons (connections) through a continuous but sometimes non-derivable line. Its shapes vary depending on material, object's geometry and potential outside constraints.

A crack appears when the internal stresses (tensions) of a material are greater than the material resistance: A

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solid liaison first breaks under the stress, then the neighbor, and so on, like springs would do in a domino fashion.

Then what is a *stress*, and what is the difference existing with a *stress-field*? Stresses arise as the material deforms. We define one stress as being the set of multi-directional tensions that exist in a material per infinitely small unit size; it can be a compressive stress or tensile stress or both depending on the direction. A stress field is the set of stresses over an entire region, surface or 3D object (note that it is not the sum).

It is difficult to represent a stress –especially in 3D—as it can go in any direction with various intensities. In our case, we are working on 3D surface layers, which gives us an excellent advantage. The surface being much bigger than the thickness of the layer, we can assume that the dimension orthogonal to the surface can be ignored. Experimentations tend to show (but not prove) that the thickness of layers should not much exceed the cell size dimension so that the crack initialization, orientation and velocity predictability stay valid. However, this problem of resolution is solved by the local cell subdivision described in the crack model description. **Figure 1 (a)** is a possible intuitive representation of a stress at a very small material region. Remark that the tension in one direction must be equal to the opposite one (see black double arrow). Therefore the graph has a central symmetry, and is represented **Figure 1 (b).** This theoretical stress spectrum is simplified as shown in Section 3.

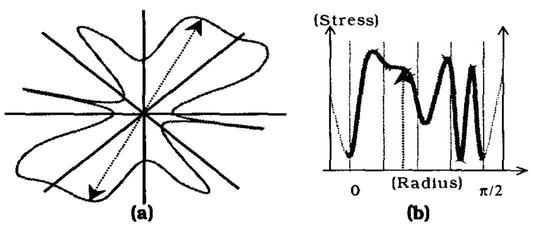

Figure 1 Stress representation per surface area **(a)** example of tension (stress) at a point and **(b)** corresponding spectrum

A strong relationship exits between stress and cracks. We think that mastering this relationship is the key to generating (and even simulating or predicting) any type of cracks on any type of material. This paper does not pretend to solve this very complicated physics problem, but proposes an initial attempt to simulate very roughly and intuitively this problem, the recursive function between crack and stress (as schematized **Figure 2**).

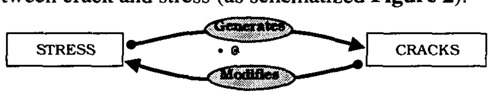

Figure 2 Main problematic: the relationship between stress and cracks

For further explanations of the physics of cracks, please refer to *Elementary Engineering Fracture Mechanics* [2], *Mechanical Behavior of Ceramics* [7],

3 Stress model.

Our model for generating crack patterns is based on a "semi-realistic" crack behavior approach. We consider it "semi-realistic" because it uses simple physical and material properties, keeping in mind the realities of the computer. The domain where the cracks evolve is a multi-layer 3D surface CA –described in [13]. The crack simulation can therefore be applied to any 3D object that can be simulated with layers, i.e., the input object has an inside volume that this crack model doesn't interact with, and its layers are non-null. Most objects satisfy to this definition (e.g., any revolution object, most ceramics and metallic object covered by painting), however applying our model to, for instance, a piece of rock is not possible, for the result becomes inconsistent.

We logically begin the description of our model by the presentation of stress, as it is the cause of the cracks. The following section details our model for simulating the stress that occurs on the 3D surface layers.

After briefly showing the reasoning used to derive the concept of *Stress Spectrum*, we describe our stress model structure in Section 3.1: how it is pre-computed and what is the need for, and relation between, material elasticity and the stress field. Then –in Section 3.2—we present a solution for the generation of cracks by stress.

Keeping in mind that at the same time the stress representation should follow the CA grid –for low computational cost—, and allow multiple stress directions, we found a convenient model that we call *stress spectrum*, which is described in the next paragraph.

3.1 Stress spectrum.

As mentioned, this model of *stress spectrum* is set on a 3D surface CA and we assume that for each cell of a surface layer, a stress field is defined. To minimize the amount of memory and optimize the stress intensity computation, and as the surface for each cell is a square, the 8 Cartesian directions define the stress spectrum (see **Figure 3 (a) and (b)**).

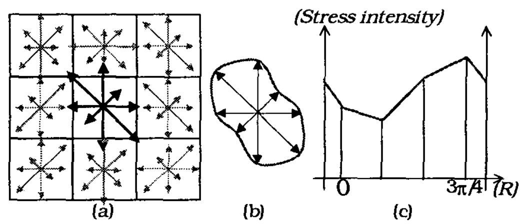

Figure 3 Stress Spectrum graphical representation **(a)** one cell, one stress **(b)** stress field **(c)** stress spectrum

In the following paragraphs, we first describe how to compute the stress spectrum and detail an intensity problem concerning surface discontinuity, and then we present a simple and efficient method for simulating the material elasticity property.

Stress spectrum intensities computation.

In nature, most of the cracks do not appear with shock or collision. For example, art's ceramics will generate their typical cracks patterns due to strong differences in temperature; clays as well as paintings will do so, due to the drying process. That is why external forces are not taken into account directly, and the stress is computed depending of the surface geometry and the material.

The assumption we make is that each of the stress directional intensities is proportional to the difference of opposite cell layer thickness and to the layer curvature in this direction. We can immediately notice that for such a spectrum model only four angles are required – from 0 to π: 0, π/4, π/2, 3π/4.

3.2 Average stress spectrum.

As we will see in Section 4, cracks have to be much more precise than cells, thus, to avoid sudden changes of stress intensity, we need a method to know the stress field not only at a cell region but also at any position between the cells. We assume that the stress field is linearly continuous which allows us to compute the average stress by a bilinear interpolation.

We have defined the stress field at any position of the 3D multi-layer CA. From this solid basis, we can (Section 4) detail how the cracks are generated on the object surface.

4 Crack modules (CM).

Crack propagation is determined by the systematic stress release of all the *unstable* cells of the input object. We define as "unstable cells" the ones that contain at least one stress directional intensity stronger than the material resistance m_R. To make the crack pattern even more realistic, the size of the crack depends on the stress release.

Crack "birth".

The initial step consists of making a list of all the unstable cells arranged in decreasing stress intensity order. All these cells try to make a crack, but sometimes a priority crack releases some neighboring region that is also listed as a weaker potential crack. That is why before selecting a new crack from the list we must first reorder the list.

As a crack can develop in multiple directions, to every crack is associated a set of what we call "crack modules" (CM). A CM is one of the crack's heads, e.g. when tearing a piece of paper only one CM is produced. In our case a crack cannot be generated with a single CM as we are working on a surface that has no beginning or ending. Note that we can conclude that it is not possible to simulate the tearing of a piece of paper with our model! In fact, as we previously said in the introduction of Section 3, a filled object cannot be simulated, and it may be surprising but indeed a piece of paper is a filled object!

We assume that only four types of crack exist that are called "I", "T", "Y" and "X":

- "I" being the most common, it consists of 2 CM's, that are initially propagated in the opposite direction;
- "T" crack type consists of 3 CM's, with 2 of them similar to the "I" type, and the last orthogonal to the others;
- "Y" crack type consists of 3 CM's too, but their initial directions form regular angles of 120°;
- "X" type is extremely rare and consists of 4 CM's with orientations form regular angles of 90°.

4.1 CM movement.

Each CM moves on its material layer –releasing the stress orthogonal to its path (Section 4.2). Its first orientation follows the higher stress release. For every time-step each CM is influenced by the current surrounding stress spectrum that is computed using the method explained Section 3.2. It then changes its orientation or even sometimes subdivides itself, generating a fork as shall be seen later.

This subsection details the individual development of a CM, which induces the desired crack path. We first define some general movement rules, then explain what we call the "kinetic potential" of CM's. From this basis, we can describe a CM's initial direction and change of orientation, its movement and finally the storing of its path in the data structure of the CA.

CM movement rules.

Here are the main rules for a correct development of a CM:

- CM's maneuver on the surface of the multi-layer cellular object;
- CM's always stay on the same layer, but as the surface can have holes (see CA [14]) different CM's can be propagated through different layers at the same time (this usually happens when the two layers have similar property);
- In their movements, CM's set the encountered cells to a cracked cell status;
- CM's are terminated if:
  - * They encounter a **new** cell which already has a cracked status;
  - * Their "Kinetic potential" reaches zero (see next paragraph);
- CM's move with a step that must be inferior to the cell size.

CM kinetic potential.

We attribute a *kinetic potential* to each CM. This property allows CM's not to stop –and therefore be terminated—on any minor obstacle. Note that this gives the curve a smoothness that is difficult to control. It becomes either an advantage for the crack simulations of many ceramics (smooth or linear propagation), or a disadvantage for material crack simulation, such as mud –requesting very irregular cracks.

This kinetic potential is simulated by an accumulation of stress. We attribute directly a certain percentage p of the maximum stress from the encountered cell's stress intensity. When this kinetic potential is below a certain threshold (about zero), the crack module stops.

CM orientation.

In the stress relaxation pre-computation section we introduced the layer's elasticity property. In this paragraph we present the primary material property that makes CM change from one orientation into another: the tolerance to stress.

All materials have a tolerance to stress, allowing for each unit surface –in our case each cell– a maximum tension to exist on any direction. If this limit is exceeded in one or more directions, it must be released so that the object becomes "stable" at this cell: cracks are generated.

The following figure shows the stress spectrum located in two different materials – the red circle showing the resistance of the respective material and the red double arrows pointing at the maximum locals:

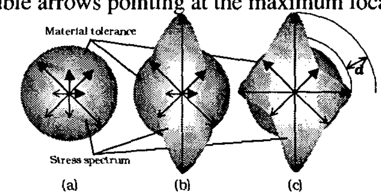

Figure 5 Material tolerance vs. Stress spectrum

In the **Figure 5 (a)** none of the stress spectrum (red) intensity is superior to the material tolerance (blue). In this case, the corresponding cell cannot generate a crack; CM's can go through it, but their kinetic potential decreases. In **Figure 5 (b)** the stress is superior to the material tolerance in one direction. This cell shall generate a crack if its stress intensity is not reduced by a neighboring crack. **Figure 5 (c)** presents a case where more than one stress direction is superior to the material tolerance. We assume that only the strongest one influences the CM direction.

CM paths.

The following **Figure 6** summarizes the main steps for determining our goal: the CM paths.

Schema (a), (b) and (c) show the change of kinetic potential and module orientation;

Schema (d) presents the possible set of directional influences through time due to local stress. Note that they always follow the 8 Cartesian directions;

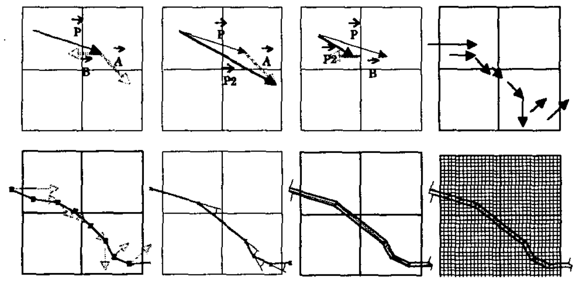

Figure 6 Determination of the crack path

Schema (e) shows the corresponding crack path, step by step; note that the resulting path does not specifically follow the 8 Cartesian directions;

Schema (f) proposes a linked-segments solution for simplifying the crack path using a simple change of angle test. The smaller the angle is, the better the resulting crack pattern, but the higher the computational and memory cost.

Schema (g) shows the possible resulting crack widths along the CM path. Here, these widths are constant but during simulation they are often variable (see Section 5, **Figure 8**)

The last schema (h) presents a possible grid superposed on the linked-segment. We later use this grid to compute an anti-aliasing simulation of the linked-segment.

The key to the generation of crack is the release of the stress contained in the material, and this is what is described in the following sub-section.

4.2 Cracks modify stress.

In the previous section we have shown how stress-fields generate crack paths. To make our model well balanced, we propose a solution to simulate the stress release surrounding CM paths.

At each sub-movement of the CM, stress is released on both side areas (hence perpendicularly to its path direction). These areas release the stress field up to a distance ***d*** with a "release intensity" dependant on the material properties, the crack thickness, and the distance between the module and the crack path (so that it linearly decreases).

This technique is illustrated in **Figure 7**, where the ideal crack path is a black oriented dot line, the CM steps are thick green segments, and the stress relaxation area (here, for purposes of simplification, its length is constant) is shown in shading from green (100%) to white (0%).

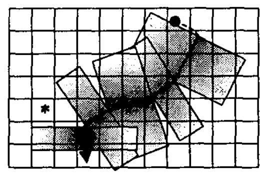

Figure 7 Releasing stress around the crack path

The interesting consequence of this algorithm is the *mutual interaction* between crack and stress: stress generates cracks, cracks modify the stress, stress then modifies cracks, etc...

5 Results.

The model being reasonably complex and as the rendering part is not the purpose of this paper, we have preferred to present here a graphical summary of the main steps we have described in this paper, proving the correct behavior and possibilities of this crack model. (Note that we are presently making the rendering implementation and that more various resulting crack patterns are presented at **http://www-cg.cis.iwate-u.ac.jp/~ stephane**).

This section is organized as follows: The first step is the pre-computation of the stress field over the different layers of the CA. The second step –**Plate 1**— presents the release of the stress contained in the material layers. **Plate 2** demonstrates the main property of our model with the mutual interaction of two cracks. Last two plates propose some possible visual crack pattern simulation using. On the first hand, **Plate 3** shows the crack propagation over a relatively complex object, and on the second hand, **Plate 4** presents the complex and precise mud crack pattern over the 3D surface of a simple tetrahedron.

[Please notice that all results (CA and graphical) were computed and rendered on a SGI Indigo2™ workstation, with 175 MHz CPU R10000, and 128 MB RAM]

Stress pre-computation & stress release.

Plate 1 is the actual stress release of a region stress field due to the movement of a CM. The first image (a) is the initial stress field. Images (b) to (e) are four steps (over 9 in reality) of the CM animation, and the last image (f) is the elimination of stress spectrum inside a 100% crack region.

As the CM exists as a visible entity, we have superimposed on the scanned pictures some interesting data. The red linked-segment is the possible crack path in this direction; the red dots on the stresses indicate whether or not stress spectrum have been affected by the release; blue rectangles represent the main areas where stress is released parallel and inversely proportional to the distance to the crack path.

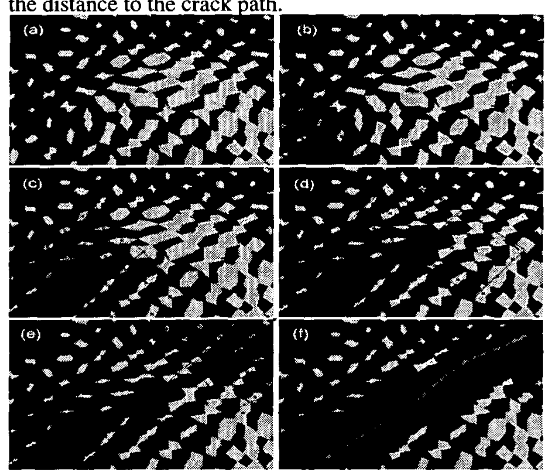

Plate 1 Progressive stress release parallel and inversely proportional to the distance to the crack path

This method is essential to determine how the stress field is modified, as illustrated by the evolution from image

(a) to (e). Note that in (a) the set of stresses is more or less continuous in all directions making a harmonious field, while in (b) residual stresses near the crack are parallel to the path. Later (see **Plate 2**), because of this parallel arrangement, other CM's will change their directions in such a way that they make an angle of 90°, more or less –depending of the CM velocity.

Mutual interaction between cracks.

The last **Plate 2** demonstrates our main goal, the mutual interaction between cracks on one facet of a tetrahedron with regular stress field (pulling 90% up-down and 10% left-right). **Image (a)** shows two cracks of type "I" regularly releasing stresses on both of their sides.

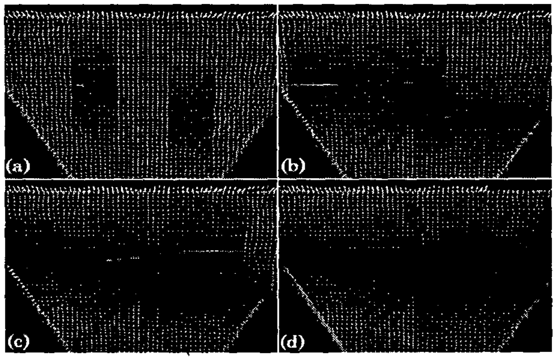

Plate 2 Mutual interaction between two parallel "type I" cracks on a regular stress field

When the cracks move toward each other (see **Image (b)**), their respective orientations (and then velocities) are modified. In image (c), the bottom crack is more and more rapidly attracted to the above one generating more or less a right angle. At the same time the above crack, which was first driven away, returns to its main course (driven toward the main intensity of the stress field). The last picture shows the final crack pattern. (Note that in Plate 2, the cyan region indicates the automatic anti-aliasing micro-cell.)

Final Results.

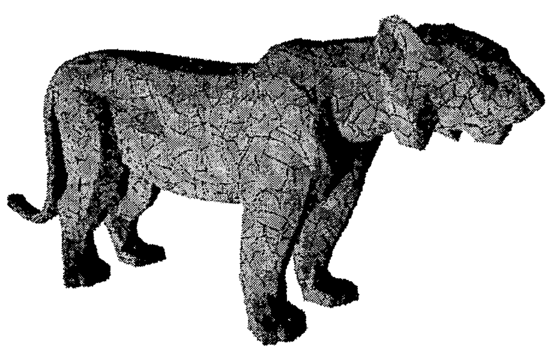

Plate 3 Crack propagation over a fairly complex object

The following **Plate 3** presents the crack pattern simulated over the first layer of a cyan ceramic tiger. As previously said, the final rendering is not shown yet, only data are proposed. Main data during this simulation were: The 1908 triangle tiger was decomposed –this time—into 197282 cells, each cell being able to be subdivided into 9x9 micro-cells (if cracked), the elasticity of the material was of 20% on a maximum range of 15 cells, and the CM's were influenced by 90% of inertia. Note also that the stress field was irregular and that the total stress released was only about 35%, and that mainly type "I" cracks were generated and propagated very straightly.

The last image –**Plate 4**—proposes the visual simulation of mud crack pattern over the 3D Surface of a tetrahedron. This approximately 330 thousands cell-simulation was obtained by setting the micro-cell grid to 11x11, 85% of the cells were initially unstable, the stress relaxation was only 5%, the elasticity recursion of 25 cells, and finally the CM's had a 65% inertia influence. The object reached 99% of stability after 20 minutes of computation. An interesting factor on this simulation is that we set the crack-seeds to be only of "type I" cracks, and yet our model naturally simulates many "Y" intersections.

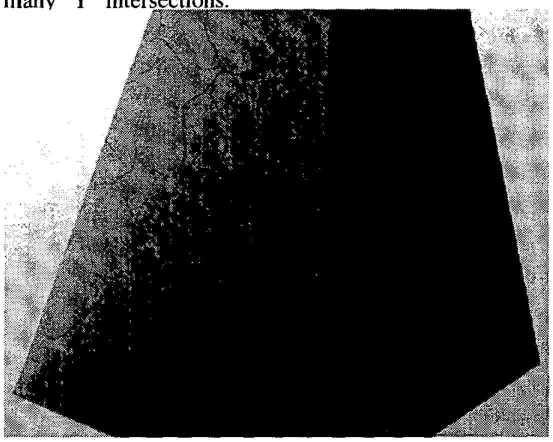

Plate 4 Visual simulation of mud crack pattern over a tetrahedron only using "type I" cracks

Discussion.

We presented an expedient model for generating crack patterns on any type of 3D object using a method based on multi-layer CA. We explained the need for a half-physical approach and proposed an intuitive but indeed efficient stress spectrum model to simulate and compute easily the stress field over any layers of the 3D surface Cellular Automaton. Then we detailed how crack patterns were generated using the stress released by crack modules. To easily improve the visual aspect of the resulting images, we have shown an original linked-segment anti-aliasing method. We finally verified our model's performance by presenting the resulting crack patterns of the four main steps used to generate the 3D surface crack propagation based on CA. We are currently working on the rendering implementation of the 3D cracks especially the intersection, facet's edge connection, and light and mirror effect for special material such as ceramics.

One precise and one general category exit in further improvement of the presented study: better multi-layers crack models and better 3D surface CA.

Many aspects remain in to improve layer cracks model. As example, make high quality crack patterns (see super-sampling [30]), or including the interactions between layers, or sorting all different crack types that current model can range.

Improving 3D surface CA will probably lead to the study of linking this 3D surface CA model to non-grid based models, such as a water flow system (as suggested in [9]). This association of models would generate realistic corrosion and the corresponding patina paintings, after the peeling of a covering layer previously cracked by the model summarized in the paper.

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